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AN APPROXIMATE MODEL OF AN ELECTRIC ARC IN TRANSVERSE MUTUALLY PERPENDICULAR AERODYNAMIC AND MAGNETIC FIELDS

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UDC 537.523.5

Expressions are obtained for the characteristics of an electric arc in transverse, mutually perpendicular, aerodynamic, and magnetic fields.

There are quite a few reports devoted to the investigation of an electric arc balanced by aerodynamic and magnetic forces or moving under the action of an external magnetic field. Among them, however, there are few theoretical solutions which admit of practical use. The concept of the impermeability of the core of an arc [1], strengthened in such a way that the arc column itself is treated as a solid cylindrical body, can therefore turn out to be admissible. The experimental investigations of the wake, the frequency of coming off of vortices, the heat exchange, and the resistance presented in [2] indicate that the flow over the core of an arc is similar to the flow over a heated cylindrical body. Indirect considerations about the high viscosity and low density of the arc column in comparison with the envelope also point to the impermeability of the arc. The gas density in the central core comprises several percent of the density of the free stream, so that when the velocities in the core and in the stream do not differ too much about 10% of the gas can pass through the arc.

The model of an arc as a solid cylindrical body naturally requires a cautious approach, since the real physical picture will differ from this idealized scheme.

We note that for such a model as a whole it doesn't matter whether the arc is under steady-state conditions with equality of the resultant aerodynamic and magnetic forces or moves under the action of the magnetic field. In the latter case the velocity, except for the initial period, must be set in such a way that it corresponds to a balance of these forces.

We assume that the properties of the arc do not vary in the axial direction, although it is well known that its characteristics also depend on the interelectrode distance. Such a dependence is primarily due to the influence of the arc sections near the electrodes. In the middle part of the arc column the variations of its properties are slight [2].

Let us write the equation for the energy balance in the electrically conducting zone of the discharge, allowing only for conductive heat transfer in the radial direction.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS}{dr}\right)+\sigma E^2=0, \quad (1)$$

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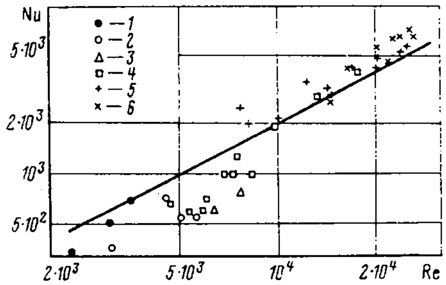


Fig. 1. Dependence Nu = f(Re) for an electric arc in a magnetic field: 1, 2, 3) V = 5.8, 8-12.2, 15.2 m/sec; I = 200-400 A [2]; 4, 5, 6) V = 32-68, 90-123, 140-220 m/sec; I = 50-550 A [4]; curve) Nu = 0.2 Re.

the equation of balance of the resultant forces at the surface of the electrically conducting cylinder of radius r_*,

$$IB = c_x \rho_\infty V_\infty^2 r_* , \tag{2}$$

and Ohm's law,

$$I = 2\pi \int_0^{r_*} \sigma E r dr. \tag{3}$$

The complete system of equations must also include Maxwell's equations and the dependences of the transfer coefficients on the temperature and pressure.

We take E = constant and σ equal to some constant mean value $\bar{\sigma}$. In this case the solution of Eq. (1) with $S_{r=0} = S_{00}$ and $(dS/dr)_{r=0} = 0$ will be

$$S = S_{00} - \bar{\sigma} E^2 \frac{r^2}{4} . \tag{4}$$

The condition $S_{r=r_*} = S_*$ gives the expression for determining E,

$$E = \frac{2}{r_*} \sqrt{\frac{\Delta S_{00}}{\bar{\sigma}}} , \tag{5}$$

while from (3) and (5) we obtain the expression for determining the current strength,

$$I = 2\pi r_* \sqrt{\bar{\sigma} \Delta S_{00}} . \tag{6}$$

From the equation curl $\vec{H} = \vec{j}$ we get the well-known expressions for the azimuthal component of the strength of the intrinsic magnetic field, by analogy with an infinite cylindrical wire containing a current. The rest of Maxwell's equations are satisfied automatically.

The radius r_* can be determined by joining the solutions of the equation of energy balance at the interface between the electrically conducting and nonconducting zones. For this one would have to solve the transfer equations for the electrically nonconducting region, which have the form of boundary-layer equations. However, one can use experimental data on the heat exchange of a cylinder in transverse streamline flow with subsequent refinement under the conditions of electric-arc burning. These data are usually represented in the form of a power-law dependence of the Nusselt number on the Reynolds and Prandtl numbers and T_*/T_∞ [3]. If

$$\mathrm{Nu} = A\,\mathrm{Re}^m \varphi, \tag{7}$$

then the average heat-flux density at the surface of a cylinder of radius r_* at the controlling temperature, equal to the temperature of the free stream, can be obtained from the expression

$$q_* = A\varphi \frac{\lambda_\infty \Delta T_\infty}{2r_*} \left(\frac{2Vr_*\rho_\infty}{\mu_\infty} \right)^m . \tag{8}$$

The condition $(dS/dr)_{r=r_*} = q_*$ makes it possible to determine the radius of the electrically conducting zone:

TABLE 1. Comparison of Equations Obtained in the Present Report and in [5-9]

Lit. source	Equation	Corresponds to Eq.
[6]	$E = 1{,}9\cdot 10^3 \sqrt[3]{\frac{V^2}{I}}$ (16)	
[7]	$V = 5{,}95\cdot 10^{-2} \sqrt{H} \sqrt[3]{\frac{I}{\rho_\infty^2}}$ (17)	
[5]	$d_* = 8\cdot 10^{-4} \sqrt{\frac{I}{V}}$ (18)	
[8]	$\frac{UL\sigma_0}{I} = 1{,}395 \left(\frac{\rho_0 h_0^2 \sigma_0^2 L^5 B}{I^3}\right)^{0,32}$ (19)	
[8]	$\frac{\rho_\infty V_\infty^2 L}{IB} = 2{,}52 \left(\frac{\rho_0 h_0^2 \sigma_0^2 L^5 B}{I^3}\right)^{0,07}$ (20)	
[9]	$\frac{E}{h_0 \sqrt{\rho_0 \mu_{e0}}} = 3{,}7\ [3{,}25] \left(\frac{\sigma_0 h_0 \mu_{e0} \sqrt{\rho_0 \mu_{e0}}\, I}{B^2}\right)^{-0,2[-0,135]}$ (21)	
[9]	$\frac{V_\infty \sqrt{\rho_0 \mu_{e0}}}{B} = 9{,}6\ [4{,}7] \left(\frac{\sigma_0 h_0 \mu_{e0} \sqrt{\rho_0 \mu_{e0}}\, I}{B^2}\right)^{0,3[0,27]}$ (22)	
	$E = 1{,}84\cdot 10^3 \sqrt[3]{\frac{V^2}{I}}$ (23)	(10)
	$V = 5{,}5\cdot 10^{-2} \sqrt[5]{\frac{H^3 I}{\rho_\infty^2}}$ (24)	(12)
	$d_* = 3\cdot 10^{-4} \sqrt[3]{\frac{I^2}{V}}$ (25)	(13)
	$\frac{UL\sigma_0}{I} = 0{,}962 \left(\frac{\rho_0 h_0^2 \sigma_0^2 L^5 B}{I^3}\right)^{0,4}$ (26)	(11)
	$\frac{\rho_\infty V_\infty^2 L}{IB} = 2{,}54 \left(\frac{\rho_0 h_0^2 \sigma_0^2 L^5 B}{I^3}\right)^{0,2}$ (27)	(12)
	$\frac{E}{h_0 \sqrt{\rho_0 \mu_{e0}}} = 4{,}1 \left(\frac{\sigma_0 h_0 \mu_{e0} \sqrt{\rho_0 \mu_{e0}}\, I}{B^2}\right)^{-0,2}$ (28)	(11)
	$\frac{V_\infty \sqrt{\rho_0 \mu_{e0}}}{B} = 5{,}6 \left(\frac{\sigma_0 h_0 \mu_{e0} \sqrt{\rho_0 \mu_{e0}}\, I}{B^2}\right)^{0,2}$ (29)	(12)

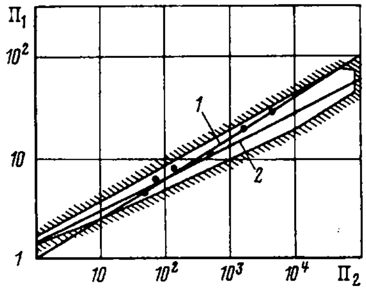
$$r_* = \left(\frac{4\Delta S_{00}}{A\varphi\lambda_\infty \Delta T_\infty}\right)^{\frac{1}{m}} \frac{\mu_\infty}{2V\rho_\infty}. \qquad (9)$$

By eliminating r_* and ΔS_{00} from (5), (6), and (9), we obtain the electric field strength as a function of the airflow velocity and the current strength:

$$E = (\pi\sqrt{\sigma})^{\frac{2-m}{2+m}} \sqrt{\frac{1}{\sigma}} (A\varphi\lambda_\infty \Delta T_\infty)^{\frac{2}{2+m}} \left(\frac{2\rho_\infty}{\mu_\infty}\right)^{\frac{2m}{2+m}} \frac{V^{\frac{2m}{2+m}}}{I^{\frac{2-m}{2+m}}}. \qquad (10)$$

One can also obtain the electric field strength expressed through the current strength and the magnetic induction. From (2), (5), (6), and (9), with allowance for the fact that the drag can be approximated by the function $c_x = C\mathrm{Re}^{-p}$, we have

$$E = (\pi)^{\frac{2(2-p)-m}{2(2-p)+m}} \left[\sqrt{\frac{1}{\sigma}} \left(\frac{2}{\mu_\infty}\right)^2 \frac{\rho_\infty}{C} (A\varphi\lambda_\infty \Delta T_\infty)^{\frac{2-p}{m}}\right]^{\frac{2m}{2(2-p)+m}} \frac{B^{\frac{2m}{2(2-p)+m}}}{I^{\frac{2(2-p)-3m}{2(2-p)+m}}}. \qquad (11)$$



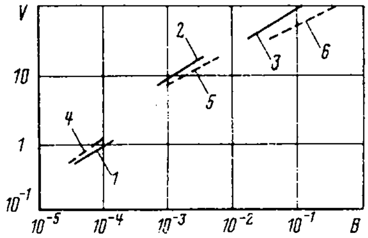


Fig. 2 Fig. 3

Fig. 2. Volt—ampere characteristic curve for the arc: 1) calculated from (26); 2) generalized Eq. (19) [8]; outlined region: array of experimental points for B = 0.012-0.108 T; L = 0.0127-0.038 m [8]; points: B = 0.0125 T; L = 0.0254 m [8]. $\Pi_1 = UL\sigma_0/I$; $\Pi_2 = \rho_0\sigma_0^2 \times h_0^2L^5B/I^3$.

Fig. 3. Comparison of calculated and experimental data on the function V = f(B, I): 1, 2, 3) calculated from Eq. (24) for I = 4, 300, and 70 A, respectively; 4, 5) experiment [2] for I = 4 and 300 A; 6) experiment [6] for I = 70 A. V, m/sec; B, T.

Equating the right sides of (10) and (11), we obtain the dependence of the velocity balancing the arc or the velocity of motion of the arc with respect to the electrodes on the external magnetic field and the current strength:

$$V = \left[\pi \sqrt{A\varphi\bar{\sigma}\lambda_\infty \Delta T_\infty} \left(\frac{2}{\mu_\infty}\right)^{\frac{2p+m}{2-2p}} \frac{C^{\frac{2+m}{2p-2}}}{\rho_\infty} \right]^{\frac{2-2p}{2(2-p)+m}} B^{\frac{2+m}{2(2-p)+m}} I^{\frac{2p+m}{2(2-p)+m}}. \tag{12}$$

The radius of the electrically conducting core of the arc is expressed through the known quantities by eliminating ΔS_{00} from (9) with the help of (6), after which

$$r_* = \left(\frac{1}{\pi^2 A\varphi\bar{\sigma}\lambda_\infty \Delta T_\infty}\right)^{\frac{1}{2+m}} \left(\frac{\mu_\infty}{2\rho_\infty}\right)^{\frac{m}{2+m}} \frac{I^{\frac{2}{2+m}}}{V^{\frac{m}{2+m}}}. \tag{13}$$

By substituting (12) into (13) one can also obtain the dependence $r_* = f(I, B)$.

Returning to (7), it should be noted that there are no experimental data in the literature on the heat exchange of strongly heated cylinders. A data analysis made in [3] shows that for laminar flow in the boundary layer the heat exchange is described by the criteria dependence (7) with the values A = 0.94-0.47 and m = 0.33-0.52, while for turbulent flow A = 0.25-0.0239 and m = 0.6-0.8. In this case the coefficient A decreases while the exponent m increases with an increase in the Reynolds number. In the tests the wall temperature of the cylinder was a maximum of 1300°K, whereas the temperature of the isothermal surface of the arc core below which the contribution of electrical conductivity becomes negligible is 6000 ± 1000°K according to the data of [2].

We determine the coefficient A and the exponent m in (7) for arc conditions by using the experimental data of [2] and [4].

The results of an analysis of these experiments in the form Nu = f(Re) are shown in Fig. 1. The Nusselt number for heat exchange between the arc and the external stream was found from the expression (ΔT_∞ = 5700°K)

$$\mathrm{Nu} = EI/\pi\lambda_\infty \Delta T_\infty. \tag{14}$$

To determine the Reynolds number when analyzing the data of [2], we used the results of a measurement of the dimensions of the arc column presented there. The dimensions were determined by photography under the same exposure conditions. The calibration showed that the measured dimensions correspond to the isotherm of 6000 ± 1000°K. The size of the arc column in the direction of the velocity vector of the oncoming stream was taken as the controlling dimension. When treating the data of [4] the column diameter was calculated from the equation [5]

$$d_* = k\sqrt{I/V}\ , \tag{15}$$

corrected with the data of [2]. Then the coefficient k proved to be twice as large as that suggested by the author ($k = 1.6\cdot10^{-3}\ m^{3/2}\cdot A^{-1/2}\cdot sec^{-1/2}$). The disagreement and the necessity of correction arising in connection with it may be explained by the different methods of defining the dimensions of the column.

The dependence presented in Fig. 1 can be approximated by the expression Nu = 0.2 Re or, taking $Pr \simeq 1$ for gases, St = 0.2. We note that the exponent to Re must not be larger than one, since in this case $St > 1$ as $Re \to \infty$.

As for the values of the drag, according to [2] one can set $c_x = C = 1$, i.e., $p = 0$.

From (10), (12), and (13) with m = 1 and p = 0 it follows that $E \sim V^{2/3} I^{-1/3}$, $V \sim B^{3/5} I^{1/5}$, $r_* \sim I^{2/3} V^{-1/3}$. Analogous results were obtained in [1] through an analysis of experimental data, equations, and dimensionless criteria under certain assumptions.

A comparison between the equations obtained from Eqs. (10)-(13) and the theoretical and empirical functions for air from [5-9] is presented in Table 1.

The values of the coefficients and exponents in the case of two-sided current supply [9] are given in brackets in (21) and (22).

In the transformation of (10)-(13) to the equations obtained by the authors of [5-9] we used the following values of the required quantities: $A = 0.2$, $m = C = \varphi = Pr = 1$, $\rho_\infty = 1.18\ kg/m^3$, $p = 0$, $\mu_\infty = 1.86\cdot10^{-5}\ kg/m\cdot sec$, $\lambda_\infty = 0.027\ W/m\cdot deg$, $c_p = 10^3\ J/kg\cdot deg$, $\Delta T_\infty = 5700°K$. The coefficients in Eqs. (26)-(29) were calculated with allowance for the values of the characteristic quantities taken by the authors of [8, 9]: $\rho_0 = 1.95\cdot10^{-2}\ kg/m^3$, $h_0 = 4.4\cdot10^7\ J/kg$, $\sigma_0 = 3\cdot10^3\ (\Omega\cdot m)^{-1}$, $\rho_\infty = 1.18\ kg/m^3$ for (26) and (27); $\mu_{e0} = 1.26\cdot10^{-6}\ H/m$, $h_0 = 3\cdot10^5\ J/kg$, $\sigma_0 = 100\ (\Omega\cdot m)^{-1}$, $\rho_0 = 1.18\ kg/m^3$ for (28) and (29). For $\bar\sigma$ we adopted its integral-mean value in that range of values of S which corresponds to the range of variation of $T = (6\text{–}20)\cdot10^3\ °K$. When using the data on $\lambda(T)$ from [10] and $\sigma(T)$ from [11] we took $\bar\sigma = 7.7\cdot10^{-3}\ (\Omega\cdot m)^{-1}$. Moreover, $U = EL$ in (26).

As seen from Table 1, in some cases the equations don't differ much from each other while in other cases the agreement is less satisfactory.

The dependence (26) is compared with the experimental data of V. Adams and with Eq. (19) [8] in Fig. 2. It is seen that whereas the entire array of experimental points is described better by (19), for the curves of B = const (26) is more suitable, although the scatter of the entire array of points will be greater than for (19).

The function V = f(B, I) calculated from (24) is compared with the experimental data of [2, 6] in Fig. 3. As is seen, the calculations and experiments agree satisfactorily.

Thus, the model discussed here for an electric arc burning in an external magnetic field and stabilized by aerodynamic forces or moving along parallel electrodes agrees satisfactorily enough with the experimental data of various authors and can be offered for practical use. The theoretical and empirical functions obtained in [5-9] can be united within the framework of this model.

NOTATION

T, temperature; $S = \int_0^T \lambda(T, P)dT$, heat-conduction function; P, pressure; ρ, density; c_p, heat capacity; q, heat-flux density; λ, μ, σ, coefficients of thermal conductivity, viscosity, and electrical conductivity; μ_e, magnetic permeability; E, H, electric and magnetic field strengths; I, current strength; U, voltage; B, magnetic induction; j, current density; V, velocity; c_x, drag; r, running radius; d, diameter; L, distance between electrodes; Nu, St, Re, Pr, Nusselt, Stanton, Reynolds, and Prandtl numbers, respectively; m, n, l, ρ, exponents; A, C, k, constants; $\varphi = Pr^n(T_*/T_\infty)^l$; $\Delta S_{00} = S_{00} - S_*$; $\Delta T_\infty = T_* - T_\infty$. Indices: ∞, free flow; *, boundary of electrically conducting zone; 00, value at axis; 0, controlling value; $-$, averaging sign.

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DETERMINATION OF THE CHARACTERISTICS OF NONSTEADY HEAT EXCHANGE IN ONE DISPERSE REACTIVE SYSTEM

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The laws of nonsteady heat exchange in a disperse reactive system are investigated within the framework of a new model of heat exchange in a disperse medium not using the concept of a coefficient of heat exchange.

The problem of energy transfer in stationary, disperse, nonreacting and reacting media has been investigated in [1-7]. An analysis of the heat-transfer mechanism is given in these reports and the basic equations are obtained. In [2], in particular, a review is made of the methods for mathematical modeling of the heat exchange in disperse media and it is pointed out that the simplest means of mathematical modeling is the use of the ordinary heat-conduction equations and the volume heat-transfer coefficients. A sufficiently simple two-temperature model for the heat exchange in disperse media is obtained as a result. A more complicated mathematical model of the transfer processes in a reacting medium, allowing for the multiphase and multitemperature nature of the medium, is suggested in [4]. In this model, however, they use the concept of the heat-transfer coefficient, which is a function of time not known in advance, and additional assumptions are introduced in connection with the use of so-called accomodation equations.

The problem of the nonsteady heat exchange in a disperse system is analyzed below within the framework of the mathematical model given in [8].

Suppose there is a vessel filled with a liquid or gaseous substance whose temperature is known and equal to T_n. A constant temperature T_n is maintained at the vessel walls during the entire process. Reactive spherical particles of a solid substance, equal in mass and having the same initial temperature T_{in}, enter the vessel at some moment. We assume that the particles will be in a suspended state at equal distances from each other and that chemical reactions whose rates are determined by the Arrhenius law [9] can be observed at the particle—gas (liquid) interface. We presume that the depletion of the material of the particles during their ignition is small, so that their depletion and the depletion of the gas is ignored. Moreover, we assume that the following assumptions are valid:

1) the number concentration of particles per unit volume of disperse medium is known and equal to n;

2) the process of heat transfer as a result of molecular heat conduction is one-dimensional;

3) the thermophysical coefficients of the particles and gas are constant;

4) the temperature inside any particle does not vary from point to point, since the radius of a particle is small.

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